Prudence Demands Conservatism [*]

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Tuesday, August 11, 2009

Abstract:

We define information systems as being conditionally conservative if they produce finer information at lower expected income levels. We then study the preference for information systems of differing levels of conditional conservatism among DM's with varying attributes including risk aversion and prudence. Similar to the Arrow-Pratt measure of risk aversion, prudence is a metric based on DM's indirect utility that measures the sensitivity of DM's' decisions to changes in risk; prudent DM's save more as income becomes riskier.

In a model of precautionary savings with information, we show that prudent DM's (those with positive prudence measures) prefer more conservative accounting systems, that imprudent DM's (those with negative prudence measures) prefer liberal accounting systems and that DM's with prudence neutral preferences (prudence measure equal to zero) are indifferent between conservative and liberal reporting systems. We next show that conservative accounting may be preferred to a perfect information system if the systems are costly. Also, we provide cases demonstrating the generality of our results, including examples where conservatism is preferred with increasing, constant and decreasing risk aversion.

1.   PRUDENCE DEMANDS CONSERVATISM – INTRODUCTION

Conditional conservatism has been described as responding to news or information in a manner that results in reporting lower income, but deferring the recognition of higher income; in the extreme case, this has been interpreted to mean "anticipate no profits but anticipate all losses."[1] Conditional conservatism has generated a lot of interest which in turn has resulted in a lot of research being done on the topic. Despite our familiarity with conditional conservatism, there remains a dearth of formal explanations for the existence and preference for conditional conservative accounting methods; we know they exist and we feel practical accountants like or prefer these methods, but we do not have a simple, formal model to explain why. Our objective is to develop such a model.

We define information systems as being conditionally conservative if they produce finer information at lower income levels. We measure prudence as the sensitivity of expected utility maximizing decision makers (DM's) to the variability of a decision variable, e.g., precautionary savings, to risk. More specifically, similar to the Arrow-Pratt measure of risk aversion, prudence is a metric based on a DM's indirect utility (i.e., the negative of the ratio of the second and third derivatives of this function) that measures the sensitivity of a DM's decisions to risk; prudent DM's save more as income becomes riskier. Using these measures, we derive the following results.

First, in a model of conservatism with precautionary savings, we show that prudent DM's (those with positive prudence measures) prefer conservative accounting systems to liberal accounting systems. Second, we show that DM's with prudence neutral preferences (prudence measure equal to zero) are indifferent to the conservative or liberal reporting systems. Third, we show that imprudent DM's (those with negative prudence measures) prefer liberal accounting systems. Fourth, we show that conservatism is preferred by prudent DM's over an accounting system that fully recognizes income based

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[1] See Watts (2003a).

on perfect signals, if the per signal cost is sufficiently high. Last, we provide cases demonstrating the generality of our results, including situations where conservatism is preferred with increasing, constant and decreasing risk aversion. These results demonstrate that decreasing risk aversion is not required for conservatism to be preferred nor does risk aversion alone suffice to explain the demand for conservative accounting methods; only prudence demands conservatism.

2.   BACKGROUND AND LITERATURE REVIEW

Recent studies offer differing definitions of accounting conservatism and argue that conservatism is valuable for different reasons.[2] However, most of these studies do not explicitly model how conservatism in accounting creates this value. We attempt to explicitly model what has become known as "conditional conservatism" and characterize how this conservatism creates value under fairly general conditions.

2.1   General discussion and definitions

Watts (2003a and 2003b) review notions of conservatism in accounting. He gives reasons why conservatism is prevalent in accounting and gives alternate definitions. The main one offered is: "higher degree of verification is required for recognition of profits versus losses". The explanations he suggests for conservatism are contracting, shareholder litigation, taxation, and accounting regulation and expects the the first two to be most important. He also discusses the role of earnings management though it is not a prime explanation. In the contracting explanation, conservatism will ameliorate managerial behavior that is of self-interest and will offset managerial misreporting. The second explanation is that by understating results, conservatism reduces the litigation costs expected by the firm. We do not use any of these reasons given for conservatism in

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[2] Watts 2003a and 2003b provide descriptions of the various definitions and a survey of the relevant literature up to 2003.

this paper, though the notion of "timeliness" offered by Watts (2003a and 2003b) is relevant for this paper. In this paper we use an alternate reason – DM's demand conservatism as it gives better information for their consumption-savings decisions. In addition to the contracting parties, we show that DM's will also prefer conservatism in reporting when making simple, general decisions, such as investing or consuming.

Building a precise economic model of verifiability and timing of recognition is somewhat tricky. Is verifiability a choice variable, is it chosen ex-ante or ex-post, is it chosen after observing some outcomes, and if so, what outcomes are relevant? Further, in any market with rational market participants, DM's will still properly infer information concerning signals that they do not directly observe. For example if a firm does not recognize a loss when the accounting method requires losses be recognized, the market will assume no loss existed. Instead the market will assume there were profits, but that the firm did not recognize these profits for some reason. In this study, we construct a model of accounting systems that are chosen ex ante and that are well defined and relatively simplistic in their structure in order to focus on the conservatism versus liberalism trade-off.

Debt contracting has been used most often as the reasons for demanding conservatism. Debt holders will trigger default if performance is bad and call the loan (Beneish and Press 1993). Further, debt holders will attempt to restrict managerial actions so as to increase the value of assets; in this attempt, conservative accounting rules will be enforced. This shareholders-bondholders conflict is shown by Ahmed et al. (2001) to lead to conservatism as an efficient contracting mechanism

If information acquisition and disclosure costs zero, DM's will prefer to have all the information. In reviewing the development of theory relating to conservatism, Watts (2003a and 2003b) points out "If information is free and there are no agency costs, then there is no role for accountants or financial reports. Accounting and reporting exist because of such costs." This paper's model shows that if full information setup is costly

then DM's will, in some cases, prefer a conservative accounting system over both a non-conservative (liberal) system as well as over a system of full recognition.

Most modern definitions of conservative accounting originate from Devine [1963] who defines conservatism as prompt revelation of unfavorable circumstances and reluctant revelation of favorable circumstances. If DM's have an asymmetric loss function, with bad news affecting the DM's' utility more strongly than good news, conservatism will be preferred. We generate an intrinsic reason for such functions using the need for precautionary savings.

2.2   Agency theory based models of conservatism

Kwon et al. (2001) use limited liability arguments to motivate conservatism. With limited penalties in a principal agent setting, they show that conservative reporting is more efficient in motivating agents. In a related paper Kwon (2005) shows that the principal can implement better effort choices by the manager with conservatism in reporting.

Gigler and Hemmer (2001) show that more conservatism lowers levels of the risk-sharing benefits derived from timely disclosure. In this paper we only show that conservatism in accounting is better than liberalism; conservatism is definitely less valuable than full recognition with perfect signals, if they are of same cost. However, since conservatism in our model results in fewer signals, we show that conservatism may be more valuable than a system that provides full recognition of perfect signals if they have the same cost per signal.

Bagnoli and Watts (2005) use managerial discretion in choosing conservatism as a signal of firm value. The market can use management's reporting policy choice to infer management's private information. However, their model differs from ours in how they construct the accounting systems. Our model does not allow for bias in the reporting, as our accounting system is based on a coarsening of the underlying state space, while their

system introduces noise into the signals depending on whether the good or bad signal is reported. While we conjecture that our results may extend to their definition of conservatism, this remains an open question.

Raith (2009) models a two period principal-agent model and shows that the manager will be paid in the first period at "conservative" (lower) rate for the first period outcome. Though the model setup (agency problem) and definition of conservatism are quite different from this paper, it has the element of saving and consumption over two periods, albeit with exponential utility functions. However Raith conflates the formal notions of prudence and conservatism in stating "The theory supports the intuition that conservatism as prudence is a response to (symmetric) uncertainty about future cash flows". In this paper we keep them separate; prudence is characteristic of DM behavior, while conservatism is a property of accounting systems. Conservatism is a solution for a prudent DM's choice of an accounting system.

2.3   Debt contracting based models of conservatism

In debt contracts setting Gigler et. al. (2007), show that conservatism in accounting is <u>less</u> efficient than the alternatives. They define conservatism through probability of reporting a high report when the true economic cash flow is low. Wrong liquidation decisions that cause inefficiency are more likely with conservatism in their model. In this study of the accounting information, we consider a model that has no conservative bias, but one that does have different levels of accuracy for good versus bad outcomes. Li, Ningzhong (2008) also uses debt-contracting to analyze accounting conservatism as asymmetric timeliness in recognition of losses and gains for unverifiable information. Li, Jing (2008) looks at the impact of accounting conservatism on the efficiency of debt contracts, and shows with high costs of renegotiation, conservatism decreases efficiency.

Guay and Verrecchia (2007) model a potentially informed manager disclosing only high private information in a setting where the market is not sure whether the manager is informed. They define conservatism as a reporting system where low firm values are reported at their actual realization, whereas high firm values are reported "conservatively", close to the definition of conservatism used in this paper. They show conservatism forces all informed firms to release the information and substitutes for commitment.

3. PRIOR RESULTS, DEFINITIONS AND MODEL DEVELOPMENT

We want to analyze how accounting information systems differ by their relative conservatism and show that prudent DM's prefer conservative accounting systems to liberal ones, but first we need to introduce some preliminaries. In the first subsection, we provide background notation and basic definitions. In the second subsection we summarize the precautionary savings problem and review some prior results related to this problem. In the third subsection, we introduce the precautionary savings problem with information, which includes developing the model of different accounting information systems.

3.1 Background notation and definitions

We start by introducing some basic notation and background definitions and then restating some important results upon which we intend to build. This model is based on the model of prudence of Kimball (1990), extended to include information systems, so we start first with the model excluding the information systems.

Let $v_j(x)$ be a utility function, defined over the consumption variable $x \in X$, for an expected utility maximizing decision-maker $j = 1,2$.[3] We assume that both DM's are

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[3] In our notation, we follow previous studies as much as possible, but variations in the notation force us to change some of the notation. We aim for consistency and choose notation closest to Pratt (1964) and Kimball (1990), when possible. We restrict mention of multiple DM's to our discussion of risk aversion.

risk averse in the sense of Pratt (1964), where DM $j = 1$ is more risk averse than a 2nd DM, $j = 2$, if for every risk, his (the 1st DM's) cash equivalent (the amount for which he would exchange the risk) is smaller than her (the 2nd DM's) cash equivalent. The cash equivalent amount that will leave the DM as well off after imposing the risk as he or she was before is called the risk premium. As Pratt (1964) shows, this is sense of risk aversion can be expressed formally as follows.

> **D1: Definition of risk aversion**: DM $j = 1$ is more risk averse than DM $j = 2$ if there exists a monotonically increasing and concave function, $g(\cdot)$, where $v_2(x) = g(v_1(x))$ holds for all $x \in X$.

The Arrow-Pratt measure of risk aversion, defined as negative of the ratio of the second and first derivatives of the utility function, was then shown to measure levels of risk aversion. More specifically, the following proposition was shown to hold.

> **D2: Arrow-Pratt measure of risk aversion**: Define the risk aversion measure as follows: $h_j(x) \equiv -\left(\dfrac{\partial^2 v_j(x)/\partial x^2}{\partial v_j(x)/\partial x} \right) = -\left(\dfrac{v''_j(x)}{v'_j(x)} \right)$ for $j = 1,2$. Then DM $j = 1$ is more risk averse than DM $j = 2$ if and only if $h_1(x) > h_2(x)$ holds for all $x \in X$.

These definitions and measure continue to form the basis for our understanding of risk aversion, with $h_j(x)$ being referred to as DM $j's$ measure of absolute risk aversion.

In addition to studying risk aversion, precautionary saving in response to risk has been studied. Precautionary savings represent the additional savings required by a utility maximizing agent if their future income is random instead of being known.[4] More generally, the focus is to study a DM's reaction to a choice or control variable that affects

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[4] See Leland (1968) for this definition.

the utility; can we characterize the interaction of the choice variable and the utility function and its effect on the DM's behavior in a manner analogous to how we measure risk aversion. Recall the study of risk aversion began by using the notion of an amount, called the risk premium, that left the DM as well off after imposing the risk as the DM was without the additional risk. In the same way, we use the precautionary saving amount to measure the sensitivity of the DM to additional risk, and it is shown that precautionary savings in response to risk is related to the convexity of the marginal utility function, or a positive third derivative of the expected utility function.

The general model starts by using the general framework for choice under uncertainty due to Rothschild-Stiglitz (1971). Assume a DM's utility can be represented by a function of two variables, a choice variable δ and an exogenous random variable θ, so that the DM chooses to maximize expected utility $EV(\theta, \delta)$. More explicitly, the DM's problem is based on the following optimization situation.

$$\max_{\delta}\{E[V(\theta, \delta)]\};$$ [Eq. 1.a]

using the first-order condition (FOC): $E\{\partial V / \partial \delta\} = 0$. [Eq. 1.b]

We will refer to the function, $V(\theta, \delta)$, as the indirect utility function of a DM. Assuming that $E\{\partial V / \partial \delta\} = 0$ is convex in θ, then increases in the variability of θ will result in increases in the optimal choice of δ. Briefly, just as the concavity of the utility function can be used to measure risk aversion, convexity of the FOC can be used to measure the optimal response of choice variables to risk.

The next step in the general model of the theory of the optimal response of choice variables to risk is to get a measure, analogous to the Arrow-Pratt measure of risk aversion, which measures the sensitivity of the optimal choice variable to risk. Such a measure, called a measure of "prudence," was offered in Kimball (1990). Prudence is described as the propensity to prepare and forearm oneself in the face of uncertainty. In

that article it was shown that the cross-partial derivative, $\partial^2 V(\theta,\delta)/\partial\theta\partial\delta$, is the key building block in the construction of the prudence measure. More specifically, if the cross-partial derivative function, $\partial^2 V(\theta,\delta)/\partial\theta\partial\delta$, is uniformly positive or uniformly negative, then define the measure $\eta(\theta,\delta)$ as follows.

D3: Definition of the prudence measure: Define the prudence measures as follows: $\eta(\theta,\delta) \equiv -\left(\dfrac{\partial^3 V(\theta,\delta)/\partial\theta^2\partial\delta}{\partial^2 V(\theta,\delta)/\partial\theta\partial\delta} \right)$.

We call $\eta(\theta,\delta)$ the absolute prudence of the DM. Absolute prudence is a good measure of the sensitivity of the optimal choice variable to risk for similar reasons that the Arrow-Pratt measure is a good measure of risk aversion. More specifically, if one DM's FOC is a concave or convex transformation of another DM's FOC, then we can characterize how the two DM's differ in their degree of sensitivity of the optimal choice variable to risk.

3.2 The precautionary savings problem and basic results

The final step in the presentation of the background material involves introducing the concrete problem which forms the heart of the initial analysis of prudence, which is the precautionary savings problem. As mentioned earlier, our model builds on the model of precautionary savings of Kimball (1990), which we then extend to include information systems, so the following is simply a summary of some of the analysis in that paper.

Consider a two period model of an expected utility maximizing decision-maker facing a decision about how much of his wealth to consume in period one. The DM has wealth that is composed of initial assets and a uncertain level of second period income and must choose how much to consume in the first period. The DM must choose how much to consume at dates 1 and 2 (end of periods 1 and 2). The DM has total beginning wealth of w_0 that is known at date 0 (beginning of period 1) and a random labor income

of $\tilde{y}$ received at date 2 (i.e., at the end of period 2). The amount the DM consumes at date 1 is denoted as c while he consumes the remainder of his wealth, $w_0 - c + \tilde{y}$, at date 2. More specifically, the DM faces the following optimization problem.[5]

$$\max_{c}\{v(c) + E[v(w_0 - c + \tilde{y})]\} \tag{Eq. 2}$$

There are two periods, with three dates, and the DM's optimization problem, variables and functions are described and defined as follows. Next, we impose additional assumptions to simplify the presentation, analogous to Kimball (1990).

Assume that the random 2nd period income is the sum of a noise variable, $\tilde{\varepsilon}$, plus a known constant, $\bar{y}$, so that $\tilde{y} = \bar{y} + \tilde{\varepsilon}$. Also, we introduce a constant, $w = w_0 + \bar{y}$, to denote the known portion of the DM's wealth. This assumption allows us to rewrite the DM's optimization problem and FOC as follows.

$$\max_{c}\{v(c) + E[v(w - c + \tilde{\varepsilon})]\} \tag{Eq. 3.a}$$

$$\partial v(c)/\partial c = E[\partial v(w - c + \tilde{\varepsilon})/\partial c] \tag{Eq. 3.b}$$

As has been noted in previous work, it is clear that the uncertainty in the 2nd period income affects the choice of the 1st period consumption in only insofar as it affects 2nd period expected marginal utility. Also, the focus shifts to the savings variable, which is defined as $s = w - c$.

Next, to put the precautionary savings problem in the general Rothschild-Stiglitz framework used to introduce the prudence measure earlier, rewrite the DM's objective function as follows.

$$V(w + \tilde{\varepsilon}, c) = v(c) + E[v(w - c + \tilde{\varepsilon})] \tag{Eq. 4}$$

Here the consumption variable c is the decision variable while the sum $w + \tilde{\varepsilon}$ is the uncertain or random variable. Using this objective, we can rewrite the analysis leading to the prudence measure shown earlier. We replace $E[\partial V/\partial \delta] = 0$ with the DM's FOC, which we write as $V' = v' - E[v'] = 0$, replace $\partial^2 V/\partial \delta \partial \theta$ with $-v''$, which is always

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[5] Kimball (1990) used the more general equation, $\max_{c}\{u(c) + E[v(w_0 - c + \tilde{y})]\}$, but we assume the same utility function for each period.

positive for risk averse DM's, and replace $\partial^3 V / \partial \delta\, \partial\theta^2$ with v'''. Since v' is constant for each fixed value of the decision variable, and since $\partial v(w - c + \tilde{\varepsilon})/\partial c$ is a function of decision variable c only through $s = w - c$, we can rewrite the prudence measure in terms of savings as follows.

$$\eta(w,c) = \eta(s) \equiv -\left(\left(\partial^3 v(s)/\partial s^3\right)/\left(\partial^2 v(s)/\partial s^2\right)\right) = -\left(v'''/v''\right) \qquad \text{[Eq. 5]}$$

This facilitates a comparison of prudence and risk aversion as we describe next.

To summarize, let $h(s) \equiv -\left(v''(s)/v'(s)\right)$ and $\eta(s) \equiv -\left(v'''(s)/v''(s)\right)$ denote the risk aversion and prudence measures, respectively. Kimball (1990), summarizing Dreze and Modiglian (1972), showed that the relative values of the prudence and risk aversion measures can be characterized in terms of the first derivative of the risk aversion measure. More specifically, we have the following general result.

> **R1 (prior result 1, equation 20, page 65 in Kimball (1990))**: Assuming the DM is strictly risk-averse with a positive risk aversion measure, then the prudence measure is greater than, equal to, or less than the risk aversion measure as the risk aversion is decreasing, constant or increasing (abbreviated as DARA, CARA and IARA, respectively), i.e., the following are true:
>
> a. If $h'(s) < 0$ holds for all s (DARA), then $\eta(s) > h(s)$ [Eq. 6.a]
>
> b. If $h'(s) = 0$ holds for all s (CARA), then $\eta(s) = h(s)$ [Eq. 6.b]
>
> c. If $h'(s) > 0$ holds for all s (IARA), then $\eta(s) < h(s)$ [Eq. 6.c]

The case of DARA is often seen as the most important case, and in this case, prudence exceeds risk aversion, which is positive. Alternatively, DARA implies the negative of the derivative of the utility function is more risk averse than the utility function, or $-v'(s)$ is more risk averse than $v(s)$.

The preceding provides the background and foundation for our analysis. Next we introduce some additional notation and definitions that pertain to the concept of conservatism as understood in accounting.

3.3   Notation and definitions for conservatism

In our model, an accounting information system involves the receipt of a signal by the DM which will be used by the DM in his or her decision making. The signal provides information about the arguments in the indirect utility function and the DM uses this information when making his or her decision. We wish to focus on the relative demand for accounting information systems, where one accounting systems is defined as conservative while a second system is defined as liberal
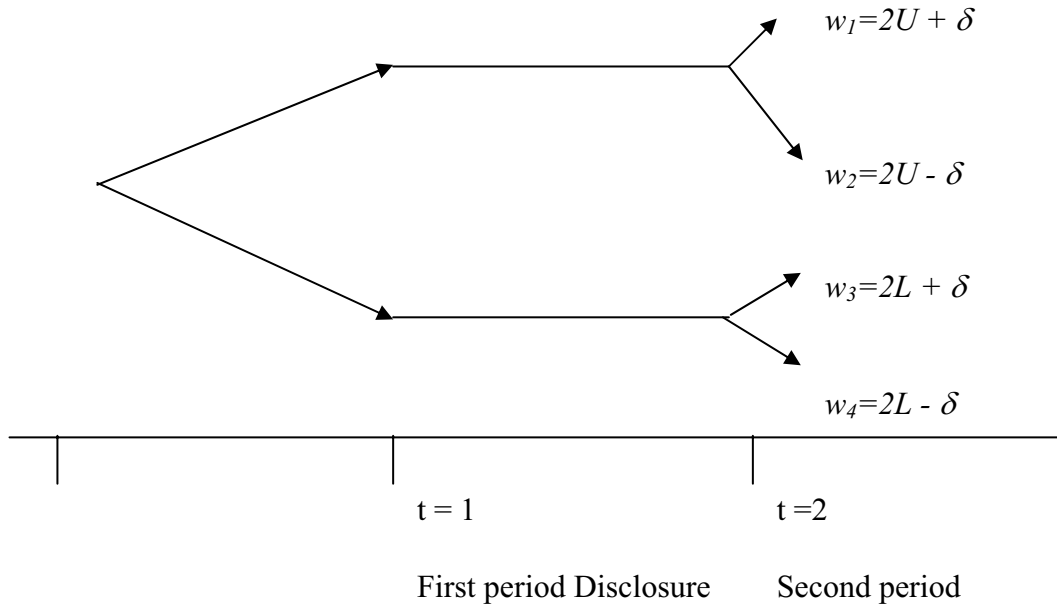
While there are many ways to describe conditional conservatism, recognizing losses while deferring the recognition of gains is included in virtually every description of the concept. In our definition of a conservative accounting system, we formalize the notion of "recognition" by assuming the signal is perfect. We distinguish between conservative and liberal accounting systems by assuming recognition occurs for the conservative system at lower levels of income and for the liberal system at higher levels. More specifically, we assume the conservative system generates perfect signals for lower levels of second period income in the precautionary savings problem, but imperfect signals at higher levels. The liberal system does the reverse. The probability distribution and the income levels are chosen to ensure symmetry in the two systems except for the distinction that we make to distinguish between conservative and liberal systems.[6] We explain our assumptions about the payoffs and the signals received from the information systems in more detail next.

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[6] We recognize that we employ a number of strong restrictions in our modeling of the information systems; we expand upon these restrictions, and their impact on our results, in our conclusion.

We assume that the DM has zero initial income, but can borrow costlessly. The DM consumes at both dates one and two, as in the precautionary savings model described above. The uncertain second period earnings, denoted as $w \in W$, takes one of four equally likely values, so that $w_n \in W = \{w_1, w_2, w_3, w_4\} = \{2U + \delta, 2U - \delta, 2L - \delta, 2L - \delta\}$, where $U > L > \delta$. The evolution of earnings is shown in Figure 1.

Figure 1: Evolution of Earnings

$w_1 = 2U + \delta$

$w_2 = 2U - \delta$

$w_3 = 2L + \delta$

$w_4 = 2L - \delta$

t = 1   t = 2

First period Disclosure   Second period

As Figure 1 suggests, earnings results can be grouped into good and bad outcomes, where good earnings are represented by the set $G = \{w_1, w_2\} = \{2U + \delta, 2U - \delta\}$ and bad earnings are represented by the set $B = \{w_3, w_4,\} = \{2L - \delta, 2L - \delta\}$.

The DM must choose how much to consume at dates 1 and 2 (end of periods 1 and 2). The DM has zero total beginning wealth and must decide how much of the random earnings of $w \in W$ received at date 2 (i.e., at the end of period 2) to borrow and consume at date 1. The amount the DM consumes at date 1 is denoted as c while he consumes the remainder of his wealth, $w_n - c$, at date 2. Before choosing his consumption at date 1, the DM observes a signal from one of four possible accounting information systems denoted as $i = Full, Part, Csv, Lib$ for Full, Partial, Conservative

and Liberal, respectively. The Full system generates perfect signals, allowing full recognition, while the Partial system generates imperfect signals, resulting in recognition based on expected values. The conservative and liberal accounting systems generate a mixed set of perfect and imperfect signals, as discussed more fully below. The signal for each system is denoted as $z_n^i \in Z^i$, where the superscript is used to indicate the accounting information system, the subscript indexes the signal and the lower case (or upper case) indicates the realized signal (set of potential signals). The set of signals in each system is discussed next, beginning with the full recognition system.

For the full recognition system, the signal is perfect so the signal set has four possible signals and is denoted as $Z^{Full} = \left\{ z_1^{Full}, z_2^{Full}, z_3^{Full}, z_4^{Full} \right\} = \left\{ w_1, w_2, w_3, w_4 \right\}$. The recognition of earnings under the full recognition accounting system is shown in Figure 2.

Figure 2: Full Recognition (perfect signals) of Earnings

$z_1^{Full} = w_1 \longrightarrow w_1 = 2U + \delta$

$z_2^{Full} = w_2 \longrightarrow w_2 = 2U - \delta$

$z_3^{Full} = w_3 \longrightarrow w_3 = 2L + \delta$

$z_4^{Full} = w_4 \longrightarrow w_4 = 2L - \delta$

t = 1
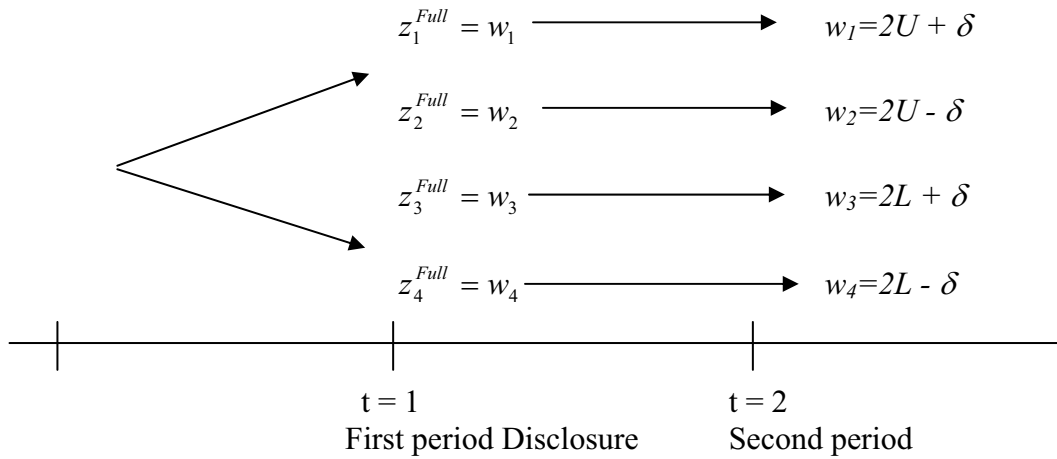First period Disclosure

t = 2
Second period

Figure 2 shows that full recognition accounting describes a situation where a perfect signal is generated. Since the second period income is known with certainty at date 1, the DM will choose to consume exactly half the income. This means that the DM optimally consumes *2U+0.5δ, 2U-0.5δ, 2L+0.5δ,* or *2L-0.5δ,* in each period, as the DM sees the signal z_1^{Full}, z_2^{Full}, z_3^{Full} or z_4^{Full}, respectively. Since there is no uncertainty, the

DM chooses a precautionary savings level equal to zero. Next we turn to the situation with imperfect signals of earnings.

For the partial recognition system, one of two possible signals is observed by the DM, where signal set is denoted as $Z^{Part} = \{z_1^{Part}, z_2^{Part}\} = \{G, B\}$. The recognition of earnings under the average recognition accounting system is shown in Figure 3.

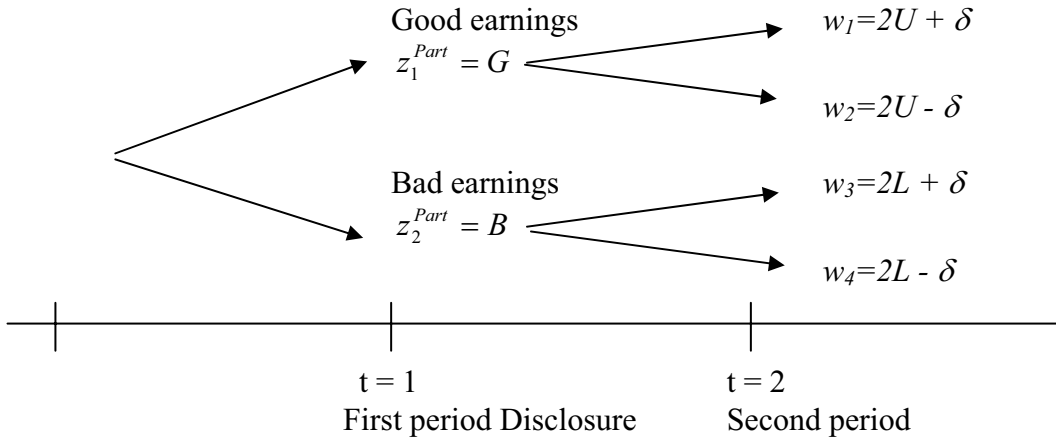**Figure 3: Average Recognition (imperfect signals) of Earnings**

Figure 3 shows that average recognition accounting describes a situation where imperfect signals are generated for good and for bad news. In both cases, the DM will choose to consume half the expected income, less some cautionary savings amount. For example, if earnings are good, the DM sees signal $G = \{w_1, w_2\} = \{2U + \delta, 2U - \delta\}$. If $z_1^{Part} = G$ is observed in the first period, the DM knows that actual earnings in the second period are either *2U + δ* or *2U - δ.* The expected earnings are *2U*, so this is also the consumption expected over the two periods. Each period's consumption on average should be *U*, but some savings (i.e., the amount that consumption is below *U* in the first period), may occur. This is what we call the precautionary savings amount; denote the amount of savings when the signal G is reported as s_G. The DM consumes $c_G = U - s_G$ in period one and, in the second period, he or she consumes whatever is generated in the second period, less the first period consumption. In an analogous manner, the DM

consumes $c_B = L - s_B$ when he or she observes signal $z_2^{Part} = B$. Since total income is denoted in general as $w \in W$, we denote precautionary savings in general as $s = w - c$.

For the conservative system, the signal set has three possible signals, and is denoted as $Z^{Csv} = \{z_1^{Csv}, z_2^{Csv}, z_3^{Csv}\} = \{G, w_3, w_4,\}$. The disclosure of earnings under the conservative accounting system is shown in Figure 4.

Figure 4: Conservative Accounting of Earnings

$z_1^{Csv} = G$

$w_1 = 2U + \delta$

$w_2 = 2U - \delta$

$z_2^{Csv} = w_3$ — $w_3 = 2L + \delta$

$z_3^{Csv} = w_4$ — $w_4 = 2L - \delta$

t = 1
First period Disclosure
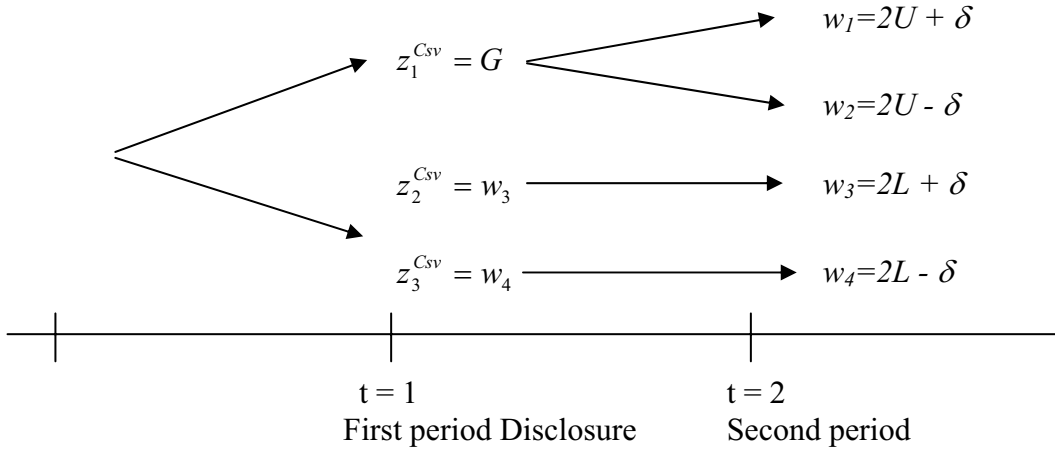
t = 2
Second period

Figure 3 shows that, under conservative accounting, the system generates the imperfect signal $G = \{w_1, w_2\} = \{2U + \delta, 2U - \delta\}$ if the earnings are good but generates the perfect signals of either $z_2^{Csv} = w_3$ or $z_3^{Csv} = w_4$ if the earnings are bad. We have designed the model so that no savings occur for perfect signals, but they may occur for imperfect ones, as we next discuss.

The total expected utility at date 1 given G is given as follows.

$$E[v|G] = v(U - s_G) + 0.5 \times v(U + s_G + \delta) + 0.5 \times v(U + s_G - \delta) \qquad \text{[Eq. 7.a]}$$

The FOC after differentiating with respect to savings is given as follows.

$$\partial E[v|G]/\partial s_G = -v'(U - s_G) + 0.5 \times \{v'(U + s_G + \delta) + v'(U + s_G - \delta)\} = 0 \qquad \text{[Eq. 7.b]}$$

The optimal savings when signal G is observed will be chosen to solve equation 7.b. If either of the bad earnings, $2L + \delta$ or $2L - \delta$, are disclosed, consumption will be half the total earnings to be realized in period 2, so $c = L + 0.5x\delta$ or $c = L - 0.5x\delta$, depending on

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which signal is observed. Hence, under conservative accounting disclosure, the total ex-
ante expected utility at date 0 is given as follows.

$$E[v|Z^{Csv}] = 0.5 \times v(U - s_G) + 0.25 \times v(U + s_G + \delta) + 0.25 \times v(U + s_G - \delta)$$
$$+ 0.5 \times v(L - 0.5 \times \delta) + 0.5 \times v(L + 0.5 \times \delta)$$

[Eq. 8]

Next we turn to the liberal accounting information system.

The liberal accounting information system is defined in a manner exactly
analogous to the conservative system, except now the perfect signals are given when the
good earnings are reported. Hence, under liberal accounting, the signal set again has three
possible signals, and is denoted as $Z^{Lib} = \{z_1^{Lib}, z_2^{Lib}, z_3^{Lib}\} = \{w_1, w_2, B\}$. The disclosure of
earnings under the conservative accounting system is shown in Figure 5.

Figure 5: Liberal Accounting of Earnings
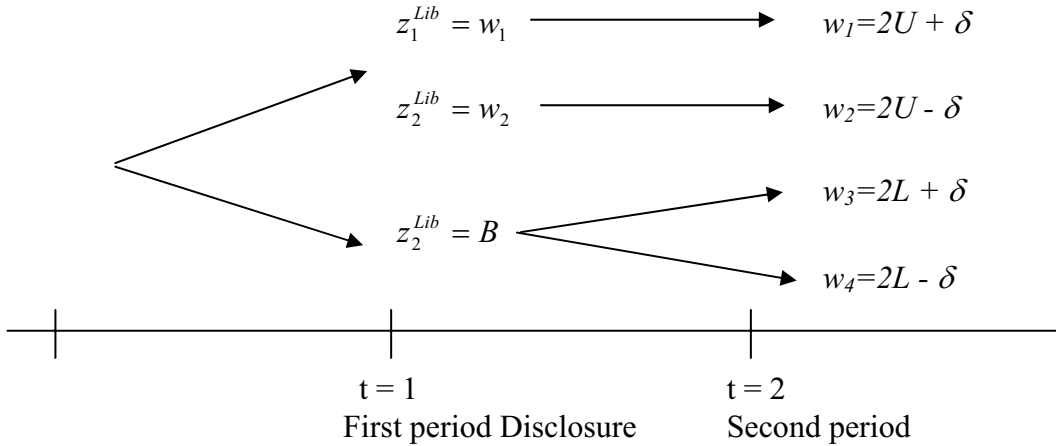
In liberal accounting, just as under conservative accounting, the signal set include
both perfect and imperfect, but now perfect signals are generated for good news and
imperfect signals are generated for bad news. As with the perfect signals under the
conservative system, it is again the case that no savings occur for perfect signals under
the liberal system, but they may occur for imperfect ones, as we next discuss.

In the first period, if $B = \{w_3, w_4\}$ is disclosed, the actual earnings in the second
period are either $2L + \delta$ or $2L - \delta$ with equal probability. Hence, the expected earnings

and total consumption amounts are *2L*, and the average consumption in each period is *L*.

The total expected utility at date 1 given *B* is given as follows.

$$E[v|B] = v(L - s_B) + 0.5 \times v(L + s_B + \delta) + 0.5 \times v(L + s_B - \delta) \qquad \text{[Eq. 9.a]}$$

The FOC after differentiating with respect to savings is given as follows.

$$\partial E[v|B]/\partial s_B = -v'(L - s_B) + 0.5 \times \{v'(L + s_B + \delta) + v'(L + s_B - \delta)\} = 0 \qquad \text{[Eq. 9.b]}$$

The optimal savings when signal *B* is observed will be chosen to solve equation 7.b. If either of the good earnings, *2U + δ* or *2U - δ*, are disclosed, consumption will be half the total earnings to be realized in period 2, so *c = U + 0.5xδ* or *c = U - 0.5xδ*, depending on which signal is observed. Hence, under liberal accounting, the total ex-ante expected utility at date 0 is given as follows.

$$E[v|Z^{Csv}] = 0.5 \times v(U - 0.5 \times \delta) + 0.5 \times v(U + 0.5 \times \delta)$$
$$+ 0.5 \times v(L - s_B) + 0.25 \times v(L + s_B + \delta) + 0.25 \times v(L + s_B - \delta) \qquad \text{[Eq. 10]}$$

Finally, we have the formal definition of our accounting information systems.

> **D4 – Definition of accounting systems**: Define the accounting systems Z^i for $i = Full, Part, Csv, Lib$, in the following manner:
> a. Full recognition accounting is $Z^{Full} = \{z_1^{Full}, z_2^{Full}, z_3^{Full}, z_4^{Full}\} = \{w_1, w_2, w_3, w_4\}$.
> b. Partial recognition accounting is $Z^{Part} = \{z_1^{Part}, z_2^{Part}\} = \{G, B\}$.
> c. Conservative accounting is $Z^{Csv} = \{z_1^{Csv}, z_2^{Csv}, z_3^{Csv}\} = \{G, w_3, w_4\}$.
> d. Liberal accounting is $Z^{Lib} = \{z_1^{Lib}, z_2^{Lib}, z_3^{Lib}\} = \{w_1, w_2, B\}$.

This clarifies the symmetry between the conservative and liberal accounting systems; next we turn to our results.

4. RESULTS

We want to analyze how accounting information systems differ by their relative conservatism and show that prudent DM's prefer conservative accounting systems to

liberal ones. In the first subsection, we show the basic result that the conservative system is preferred by prudent decision makers over the liberal one. In the second subsection, we extend the results to consider the impact of introducing cost to the accounting systems. In the third subsection, we use simple cases to demonstrate the generality of our results.

4.1   Simple model of precautionary savings with conservatism

As mentioned earlier, our model starts with the model of precautionary savings of Kimball (1990) extended to include information systems. While we have introduced the background notation in section 2, we now need to make more explicit the actual problem that is being solved and how we measure preferences. For our first step, we have the DM choose the optimal savings that solves the precautionary savings problem with information. We state this problem formally as P1 below.

P1: Precautionary savings problem with information: Define this as the problem where a DM with utility function $v(s)$, wishes to maximize the following objective function

$$\max_{s}\left\{v(s)+E\left[v(s)\big|Z^{i}\right]\right\} \text{ for } i = Csv, Lib \qquad \text{[Eq. 12]}$$

Here we define savings as $s = w - c$. The expectation is taken over the set of four equally likely uncertain 2$^{\text{nd}}$ period earnings amounts, denoted as follows

$$w \in W = \{w_{1}, w_{2}, w_{3}, w_{4}\} = \{2U + \delta, 2U - \delta, 2L + \delta, 2L - \delta\},$$

and the information system, systems Z^{i} are as defined in D4.

While problem P1 is the critical problem that our DM's solve, our real focus is on which accounting system the DM prefers. We use the expected value to measure preferences and define the preference ordering, "$\succ$", based on the relative expected utility achieved under each system. More explicitly, we define the preference ordering as follows.

D5 – Definition of preferences: A DM prefers one accounting system to another, denoted as "$\succ$", if, under the optimal choice of savings, the DM has a higher expected utility under the first accounting system than under the second, i.e., denoting the optimal savings given signal z_n^i as $s(z_n^i)$, then the follow are true.

a. $\quad Z_{Csv} \succ Z_{Lib}$, if and only if $E\left[v\left(s\left(z_n^{Csv}\right)\right)\middle|Z^{Csv}\right] > E\left[v\left(s\left(z_n^{Lib}\right)\right)\middle|Z^{Lib}\right]$

b. $\quad Z_{Csv} \prec Z_{Lib}$, if and only if $E\left[v\left(s\left(z_n^{Csv}\right)\right)\middle|Z^{Csv}\right] < E\left[v\left(s\left(z_n^{Lib}\right)\right)\middle|Z^{Lib}\right]$

c. $\quad Z_{Csv} \approx Z_{Lib}$, if. and only if $E\left[v\left(s\left(z_n^{Csv}\right)\right)\middle|Z^{Csv}\right] = E\left[v\left(s\left(z_n^{Lib}\right)\right)\middle|Z^{Lib}\right]$

Having defined the precautionary savings problem with information and how we measure the DM's relative preference for the two accounting systems, we can now turn to our results. Our main result, Theorem 1, characterizes the DM's preference between the two information systems.

Theorem 1: For all expected utility maximizing DM's facing the precautionary savings problem described in P1 above, the follow are true.

a. DM's prefer the conservative system if they are prudent or are prudent positive, i.e., $Z^{Csv} \succ Z^{Lib}$, if $\eta(s) > 0, \forall s.$

b. DM's prefer the liberal system if they are imprudent or are prudent negative, i.e., $Z^{Csv} \prec Z^{Lib}$, if $\eta(s) < 0, \forall s.$

c. DM's are indifferent between the liberal and the conservative systems if they are "aprudent" or prudent neutral, i.e., $Z^{Csv} \approx Z^{Lib}$, if $\eta(s) = 0, \forall s.$

When the DM observes a perfect signal, as he does when the information is good under liberal accounting or when it is bad under conservative accounting, the DM chooses perfectly smoothed consumption, i.e., consumption that it is the same in each period. When the signal is imperfect, as it is for bad news under liberal accounting and

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good news under conservative accounting, the DM is forced to deviate from smoothed consumption. The prudent DM saves when faced with uncertainty; in our problem, he will save and consume less in the first period than the average expected amount. The intuition for the results in Theorem 1 can best be understood by examining figures 6-8, which show the choices being faced by the DM.

Figure 6 shows the utility when news is good with both perfect and imperfect signals. Under liberalism, the good news is signaled perfectly, so the DM consumes at point E = $U + 0.5x\delta$ (or point D = $U - 0.5x\delta$) on the graph in both periods if signal $z_1^{Lib} = w_1$ (or signal $z_2^{Lib} = w_2$) is reported. The expected utility from good news under liberalism is the average of D and E, which is shown as point J in figure 5. Under conservatism, the signal is imperfect, so the DM knows are either $2U + \delta$ or $2U - \delta$, but not which one. The expected earnings are $2U$, so the DM chooses to consume A $= U - s_G$ in the first period and either B $= U + s_G + \delta$ or C $= U + s_G - \delta$ in the second period, where the savings s_G is chosen to equate the marginal utilities in equation 7.b introduced earlier. The expected utility, conditional on realization $w_2 = U + s_G - \delta$, (i.e., the utility expected from consuming at points A and C) is shown as point F in the graph while the expected utility, conditional on realizing $w_1 = U + s_G + \delta$ (i.e., from consuming at points A and B) is shown as point G in the graph. The average expected utility from good news under conservatism is the average of F and G, shown as H in the graph. Since J exceed H in utility terms, the DM has higher utility from liberalism (the perfect signal) than from conservatism (the imperfect signal) when the earnings are good.

That is the intuition for why the finer signal is preferred. The intuition for why conservatism is preferred to liberalism is that the relative value, in utility terms, of having the finer signal at lower income levels is higher than it is at higher utility levels. This works because we assume that the marginal utility function is convex, which means the concavity of the utility function is increasing. We often speak of the second derivative in terms of its effect on the first derivative; e.g., we say the concave utility function means

utility increases at a decreasing rate. Prudence means that the rate of the decrease is itself increasing. Showing how this works is a little more complicated; we use all three figures, 6-8, but especially figures 7 and 8, to accomplish this task.

Figure 6 showed that, under bad news, the expected utility from a perfect signal, as represented by point J, exceeds the expected utility from an imperfect signal, as represented by point H. The difference, J – H, relies on all the consumption points, A – E, where the DM realizes J under conservatism and H under liberalism. We have an analogous group of points for good news, e.g., A' – E', that is centered around U, and an analogous difference in utility for the perfect and imperfect signals, e.g., J' – H'. However, for good news, the DM realizes H' under conservatism and J' under liberalism. To prove that conservatism is preferred, we must show that the condition that the J – H difference decreases as the group of relevant consumption points, A – E, increases, holds; this is shown in figure 7. Alternatively, the excess in expected utility under conservatism, J – H, realized when earnings are bad, exceeds the excess in expected utility under liberalism, J' – H', realized when earnings are good. To see why prudence implies this condition holds, consider figure 8.

Figure 8 shows the marginal utility of each of the consumption points introduced in the discussion of figure 6, but using lower case, so that the marginal utility of A is a, the marginal utility of B is b, etc. First we see that the marginal utility of A, a, equals the ½ the marginal utility of C plus ½ the marginal utility of B, a = ½ c + ½ b; as mentioned, this represents the solution to equation 7.b above. Perhaps more important, figure 8 shows how the difference, J – H, changes as the income levels change.

The change in the difference J – H can be decomposed into the change in two other differences relating to the two income realizations. The first change difference relates to income $w_2 = U + s_G - \delta$ and represents the change in the difference between consuming at point D each period versus consuming at points A and C, as represented by point F. This change in difference is represented in figure 8 as the difference d – f. The

second represents the change in the difference between consuming at E each period or consuming at A and B, as represented by point G. This change in difference is represent in figure 8 as the difference e – g. Figure 8 shows that these difference are negative, so this means the change in the difference J – H will decrease, insuring that prudence implies the DM prefers conservatism.

While we have the basic result that prudence drives the demand for conservatism, this results raises other issues, such as what role, if any does risk aversion and changing risk aversion have on the demand for conservatism. We turn to some of these questions in the next subsection

4.2   Extension to the basic result on conservatism

We begin the extension of the basic result by again leveraging off of prior research. We know that risk averse DM's with decreasing risk aversion will be prudent. We this result to extend the basic result on prudence and conservatism in the following corollary.

Corollary 1.1: Under conditions of Theorem 1, if the DM is strictly risk averse and has decreasing risk aversion, then he or she prefers the conservative system, i.e., if for all savings levels, $h(s) > 0$ and $h'(s) < 0$ both hold, then $Z_{Csv} \succ Z_{Lib}$ holds.

Kimball (1990) noted that risk averse DM's who exhibit decreasing absolute risk aversion are also prudent, as shown in result R1 introduced earlier. It follows immediately that these DM's prefer conservatism to liberalism in their accounting. This result is important as risk averse investor with decreasing absolute risk aversion form a group of DM's that are arguable one of the most important groups in economic theory.

We next consider the situation where accounting systems are costless to employ. It is reasonable to assume that generating signals requires incurring costs; in the

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following analysis, we assume that the cost of a signal generating system is linear in the number of signals it generates. Under these conditions, we find that conservatism is preferred to full recognition for costs that are sufficiently high. We present this result in the following theorem.

Theorem 2: Let the conditions of Theorem 1 hold, suppose $\eta(s) > 0, \forall s$, and also now assume each accounting system costs the DM a cost of $C > 0$ per signal generated. Then there exists cut-off costs, $0 < C_1 < C_2$, such that the following are true.

a. For $0 < C < C_1$, $Z^{Full} \succ Z^{Csv}$ holds.

b. For $C = C_1$, $Z^{Full} \approx Z^{Csv}$ holds.

c. For $C_1 < C < C_2$, $Z^{Csv} \succ Z^{Full}$ holds.

d. For $C = C_2$, $Z^{Csv} \approx Z^{Part}$ holds.

e. For $C_2 < C$, $Z^{Part} \succ Z^{Csv}$ holds.

Initially, one might think this result follows immediately, however this is not the case. Even though the cost of full recognition increases faster than the cost of conservative accounting with an increase in per signal cost, comparing the relative expected utility under the two systems is complicate because the optimal precautionary savings level also changes. As the wealth of the DM decreases, with the increase in cost, the DM saves more. Theorem 2 shows that the change in the savings level does not prevent the relative expected utility of conservatism versus full recognition from rising and eventually causing it to turn positive.

We know that full recognition is preferred if $C = 0$ and Theorem 2 tells us that for sufficiently high enough cost, first conservatism and then partial recognition is preferred by prudent DM's. Theorem 1 and 2 together indicate that the preference for accounting systems is not balanced; prudent DM's prefer conservatism while imprudent

DM's prefer liberalism. One is tempted to conjecture that a balance or neutral system would never be preferred by DM's that had a non-zero prudence measure.

For example, consider the following accounting system, which we refer to as partial recognition accounting system with costly auditing. At date 1, the DM pays a cost of $C > 0$, and receives a signal reported under the partial recognition system, so that the DM knows whether good news or bad news will occur. Further, with probability $1 > \gamma_i > 0$ for $i = B, G$, an audit, or additional investigation, will reveal the perfect signal. A neutral accounting system with auditing would have $\gamma_B = \gamma_G$, so that the recognition is equally likely under a neutral system. Our conjecture is that, regardless of the cost, a DM would prefer a neutral system only if that DM had a prudence measure of zero. We formalize this conjecture in the following corollary.

Corollary 2.1: Under conditions of Theorem 1, the DM is offered an opportunity to adopt a partial recognition system with costly auditing, with perfect information revealed under good and bad news with probability and , respectively. Then DM's having a non-zero prudence measure would never prefer a neutral system. Instead, for intermediate cost levels, DM's prefer systems with higher probability on recognizing bad news or good news, as they are prudent or imprudent, respectively. I.e., $\eta(s) \neq 0$ implies $0 < \gamma_B = \gamma_G < 1$ is never optimal, while $\eta(s) > 0$ and $\eta(s) < 0$ implies $\gamma_B > \gamma_G$ and $\gamma_B > \gamma_G$ is preferred, respectively.

Corollary 2.1, which follows almost immediately using proof techniques similar to those used to prove Theorem 2, formalizes the idea that balanced recognition will not, in general, be preferred. This result is contrary to much of the current discussion in academic and regulatory circles. Most arguments are framed in terms of risk neutral DM's, for whom unbiased accounting information is preferred. Yet, as Corollary 2.1 shows, prudent DM's will not prefer unbiased accounting, at least where unbiased

accounting means that income is recognized with the same probability at both high and low income levels. Corollary 2.1 instead suggests that biased or unbalanced recognition, where either high or low income is recognized with greater probability, will be preferred. This also suggests that unbiased accounting may be more common in practice.

We next present some cases to provide intuition and insight into the relative preferences of different types of DM's.

4.3   Common utility functions and their preferences for conservatism

We use this section to discussion examples that provide insight into prudence and conservatism. We start with a discussion of some common utility functions and describe whether these functions represent DM's that are prudent. We follow this discussion by providing additional formal results that clarify how risk aversion and changing risk aversion affect preferences for conservatism. The point of these discussions is to emphasize that it is prudence, and not risk aversion or changing risk aversion, that drives the preference for conservatism.

Our discussion starts simply, as we identify whether or not some common utility functions have prudence. Many of the most common forms for the utility function indicate that DM's having these utility functions are prudent. For example, DM's who have the natural logarithm utility function, $v(x) = \ln(x)$ for $x > 0$, the simple exponential utility function, $v(x) = x^{\lambda}$ for $1 > \lambda > 0$, or the square root utility function $v(x) = \sqrt{x}$ for $x > 0$, are both risk averse and exhibit decreasing absolute risk averse (DARA). Hence, by corollary 1.1, we know they are prudent and prefer conservatism. However, prudence and preference for conservatism hold despite the fact that the relations between prudence and risk aversion differ for these three utility functions. For example, the natural logarithm utility has prudence that is always twice the level of absolute risk aversion, or $\eta(x) = 2h(x)$. The simple exponential utility has prudence that is always a constant ratio of the level of risk aversion based on the exponential $1 > \lambda > 0$, or more specifically,

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$\eta(x) = \left(\dfrac{1-\gamma}{2-\gamma}\right)h(x)$. Finally, the square root utility has prudence and risk aversion always

equal, or $\eta(x) = h(x)$. These cases indicate that our main result, that prudence implies a

preference for conservatism, holds under many relations between prudence and risk

aversion.

We next more directly address questions regarding the role of risk aversion and

changing risk aversion; questions concerning what initially may appear to be results that

drive the demand for conservatism. First, one might think that risk aversion alone will

drive the demand for conservative accounting. Second, one might think that changing

risk aversion, in particular, decreasing risk aversion, is what drives the demand for

conservatism. Two results follow, in Theorem 3, that are demonstrate that, while

prudence insures preferences for conservatism, risk aversion is not sufficient while

DARA is not necessary.

Theorem 3: For all expected utility maximizing DM's facing the precautionary

savings problem described in P1 above, the follow are true.

  a. DARA is not necessary to ensure the DM prefers the conservative system, i.e.,
     there exists a utility function where $Z^{Csv} \succ Z^{Lib}$ and $h'(s) > 0, \forall s$ both hold.

  b. Risk aversion is not sufficient to ensure that the DM prefers a conservative
     accounting system, i.e., there exists a utility function where $Z^{Csv} \prec Z^{Lib}$ and
     $h(s) > 0, \forall s$ both hold.

Probably the most commonly utilized utility function is the negative exponential.

DM's with negative exponential utility functions, $v(x) = -\lambda e^{-\lambda x}$ for $\lambda > 0$, are also

prudent and prefer conservatism. As is well known, the negative exponential utility

function exhibits CARA, so this is an example that demonstrates part a of Theorem 2.

With CARA utility, we have absolute risk aversion equal to a constant; for the negative

exponential, we also have prudence equal to this same constant, which is the parameter $\lambda > 0$. Since risk aversion does not change with changes in the DM's wealth, CARA utility such as the negative exponential function are seen as attractive in many types of analyses where we wish to isolate the wealth effects. We see from this case that even with no wealth effects (i.e., even when prudence remains unchanged with changing levels of the DM's wealth), the DM still prefers conservatism.

For part b, we show that a DM can be risk averse and imprudent, so that this DM prefers liberalism, i.e., part b holds. This occurs because the first derivative of the utility function is concave; as usual, the first derivative is decreasing (i.e., the utility function is concave), but now it decreases at a decreasing rate. Hence risk aversion does not by itself imply that the DM is prudent. In summary, DARA is not necessary and risk aversion is not sufficient; as our paper title indicates, it is prudence that drives the demand for conservatism.

5.   SUMMARY

Our research objective was to show how conservative accounting may be demanded by decision makers (DM's) based simply on the characteristics of the DM's themselves. We show that prudent DM's will prefer conservative accounting systems over liberal accounting systems where these accounting systems are equivalent in the informational sense that they provide the same level of disaggregated signals, but that the differ only in whether that dis-aggregation applies to higher or lower income signals. If the better (i.e., disaggregated or finer) information is provided for lower income signals, we call the system conservative. Alternatively, if the better information is provided for higher income signals, we call the system liberal. We find that prudent DM's, i.e., those whose marginal utility decreases at an increasing rate, put higher value (in terms of expected utility) on better information at lower income levels than on better information

at higher income levels. The reverse holds for imprudent DM's, i.e., those whose marginal utility decreases at a decreasing rate.

We see at least two important implications from our findings. First, it is prudence, not risk aversion or changes in risk aversion, that drive the demand for conservatism. Second, we show that the demand for conservative accounting can be explained by fundamental characteristics of the DM's themselves, without resorting to arguments based on contracting. Most alternative explanations for the demand for conservatism rely upon contracting related arguments that often involve asymmetric information and debt related interactions. Our model provides an argument that conservatism is valued by DM's due to their intrinsic characteristics; that is, simply because they are prudent. The key aspect of our model is that prudent DM's use the information differentially, relying more heavily on the information about lower income levels than they do on higher income levels. We feel that this fundamental manner of explaining the demand for conservatism offers a strong, hitherto unidentified, reason that conservative accounting methods are so prevalent in accounting.

We have a number of issues that we hope to address in subsequent research. First, we characterize the accounting systems in a very structured manner. On the one hand, this allows us to isolate the impact of the conservatism versus liberalism trade-off. Often one argues that conservatism involves greater bias in reporting; we believe that our characterization of the conservative and liberal accounting methods does not endow either method with greater bias, so that bias does not drive the relative preferences. However, most accounting systems are more complicated than those developed in our model. One area of further research involves relaxing the structure of the model to allow for greater flexibility in the characterization of the accounting systems.

A related topic for future research is to compare and contrast our results with other analytic results in a more formal manner. In particular, other research characterizes conservative accounting methods differently than do we. One obvious question that we

would like to answer is whether or not our results extend to these alternative definitions of conservative accounting methods.

Another topic for future research concerns potentially testable empirical hypotheses. There has been much empirical research testing for the presence of conservative accounting; our research suggests that conservative accounting should be found where prudent DM's represent a strong presence. Further, there is significant empirical research being conducted to find prudent DM's. Our results suggest that conservative accounting may offer another approach to identifying where prudent DM's may be operating.

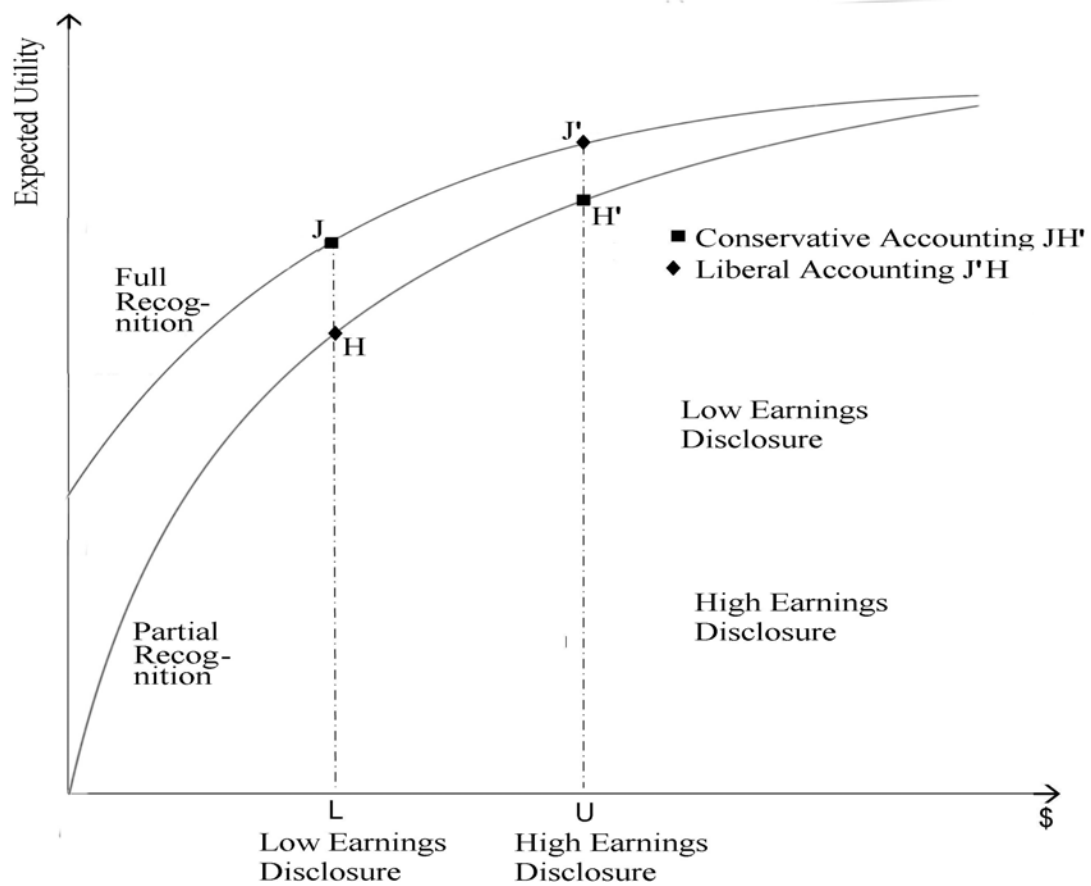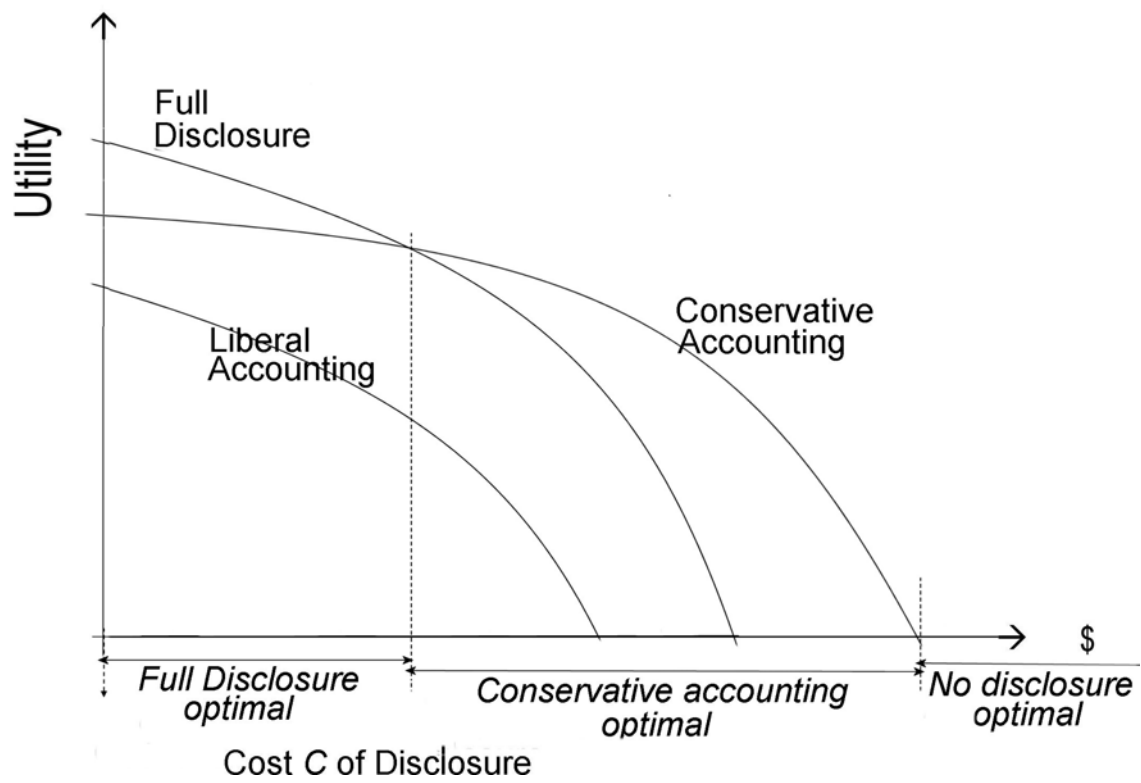**Figure 6:** Utility of consumption with perfect and imperfect signals.
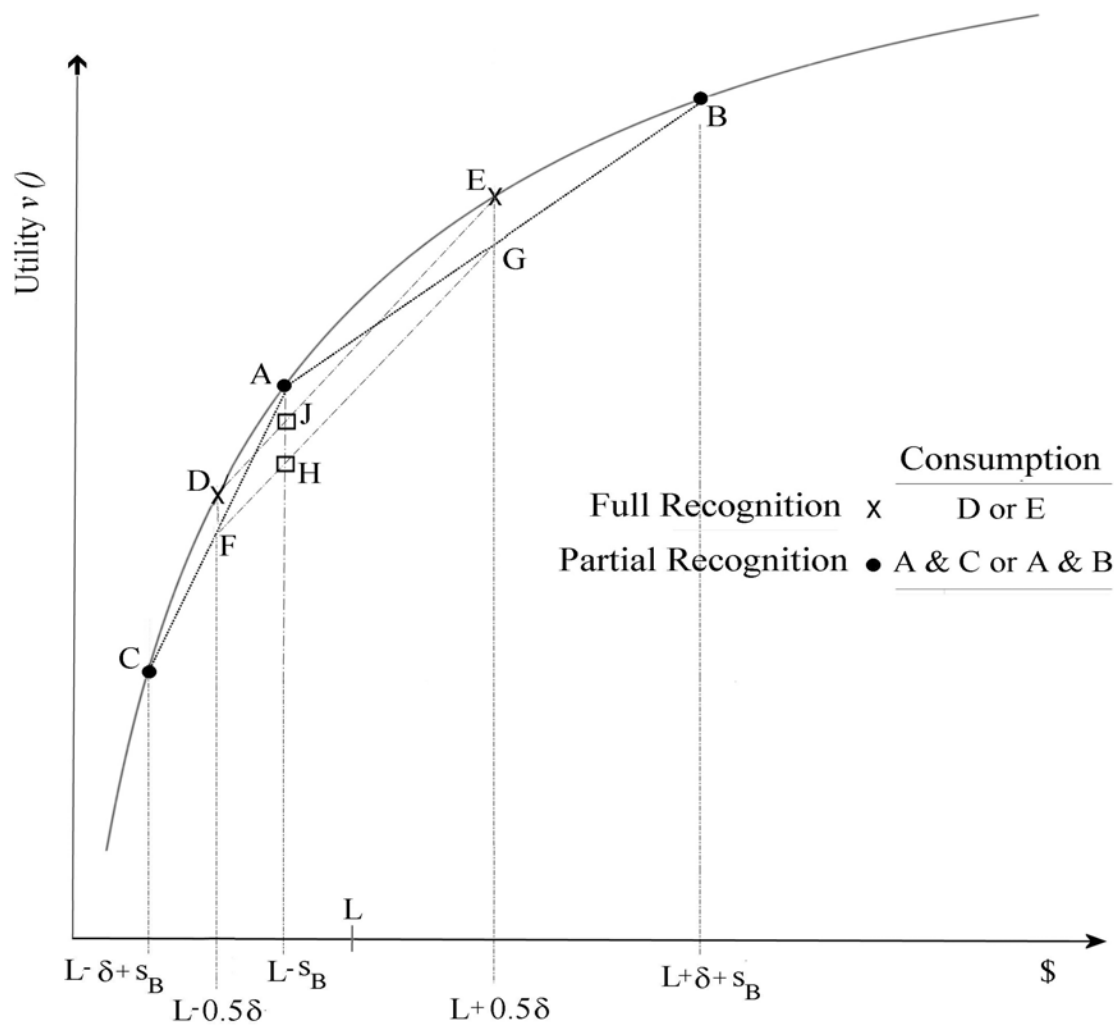
Figure 7: Marginal utility of consumption with perfect and imperfect signals.
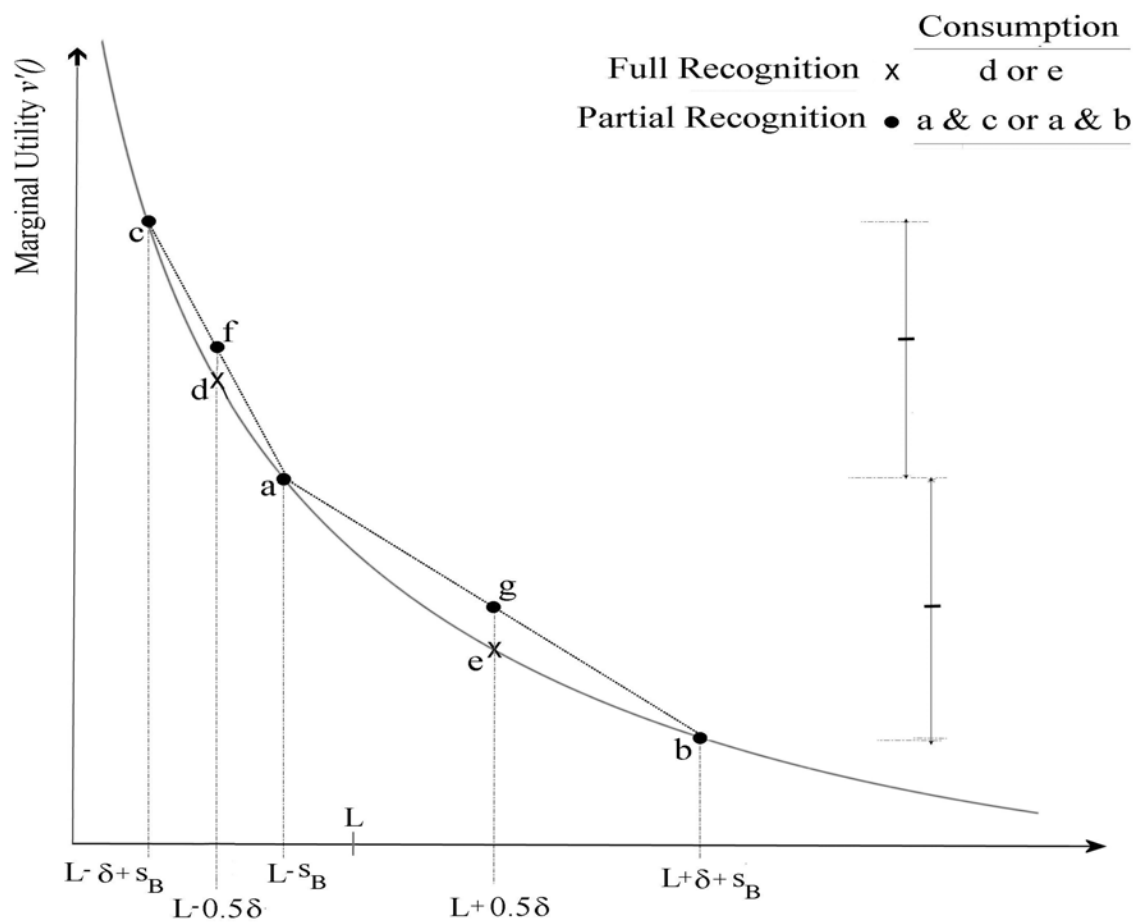
Figure 8: Expected utility of consumption with perfect and imperfect signals.

Figure 9: Consumption with perfect and imperfect signals.

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7.　　APPENDIX A: Proofs of Results

In this appendix we present the proofs of our formal results.

Theorem 1: For the information systems, Z^i for $i = Csv, Lib$, defined in definition D3 and for all expected utility maximizing DM's facing the precautionary savings problem described above, the follow are true.

　　a. $Z_{Csv} \succ Z_{Lib}$, if $\eta(s) > 0, \forall s$.

　　b. $Z_{Csv} \prec Z_{Lib}$, if $\eta(s) < 0, \forall s$.

　　c. $Z_{Csv} \approx Z_{Lib}$, if $\eta(s) = 0, \forall s$.

Proof of Theorem 1:

　　We start with the proof of part a., parts b. and c. will follow in an analogous fashion. To prove $Z_{Csv} \succ Z_{Lib}$ holds, if $\eta(s) > 0, \forall s$, it suffices to show if $\eta(s) > 0, \forall s$ implies that the DM has greater expected utility under the conservative rather than liberal accounting, or that the following inequality holds.

$$E\left[v \middle| Z^{Csv}\right] > E\left[v \middle| Z^{Lib}\right] \qquad \text{[Eq. A.1]}$$

From equations 8 and 10 in the text, the fully written inequality is as follows.

$$
\begin{aligned}
E\left[v \middle| Z^{Csv}\right] &= 0.5 \times v(U - s_G) + 0.25 \times v(U + s_G + \delta) + 0.25 \times v(U + s_G - \delta) \\
&\quad + 0.5 \times v(L - 0.5 \times \delta) + 0.5 \times v(L + 0.5 \times \delta) \\
&> E\left[v \middle| Z^{Lib}\right] = 0.5 \times v(U - 0.5 \times \delta) + 0.5 \times v(U + 0.5 \times \delta) \\
&\quad + 0.5 \times v(L - s_B) + 0.25 \times v(L + s_B + \delta) + 0.25 \times v(L + s_B - \delta)
\end{aligned}
\qquad \text{[Eq. A.1.a]}
$$

From equations 7.a and 9.a in the text, the FOC for choosing the optimal savings, s_G for conservative and s_B for liberal accounting, are shown in the following equations.

$$\frac{\partial E[v|G]}{\partial s_G} = -v'(U - s_G) + 0.5 \times \{v'(U + s_G + \delta) + v'(U + s_G - \delta)\} = 0 \qquad \text{[Eq. A.2.a]}$$

$$\frac{\partial E[v|B]}{\partial s_B} = -v'(L - s_B) + 0.5 \times \{v'(L + s_B + \delta) + v'(L + s_B - \delta)\} = 0 \qquad \text{[Eq. A.2.b]}$$

Equations A.2.a and A.2.b are used repeatedly in the following proof.

Begin by considering the problem when $\delta = 0$. As the FOC of equations A.2.a and A.2.b show, in this case the optimal savings are $s_G = 0 = s_B$ and equation A.1 holds with equality. Next, letting δ increase, we have the following equations.

$$\frac{\partial E[v|Z^{Cvs}]}{\partial \delta} = 0.25 \times \left\{ \begin{array}{l} v'(U + s_G + \delta) - v'(U + s_G - \delta) \\ -v'(L - 0.5\delta) + v'(L + 0.5\delta) \end{array} \right\} + \frac{dE[v|z^{Csv}]}{ds_G} \frac{ds_G}{d\delta} \qquad \text{[Eq. A.3.a]}$$

$$\frac{\partial E[v|Z^{Lib}]}{\partial \delta} = 0.25 \times \left\{ \begin{array}{l} v'(L + s_B + \delta) - v'(L + s_B - \delta) \\ -v'(U - 0.5\delta) + v'(U + 0.5\delta) \end{array} \right\} + \frac{dE[v|z^{Lib}]}{ds_B} \frac{ds_B}{d\delta} \qquad \text{[Eq. A.3.b]}$$

From the FOC of equations A.2.a and A.2.b, $\dfrac{dE[v|z^{Csv}]}{ds_G} = 0 = \dfrac{dE[v|z^{Lib}]}{ds_B}$.

To show that A.1 holds, it suffices to show that A.3.a exceeds A.3.b, or, rearranging and simplifying, it suffices to show that A.4 holds, where A.4 is given as follows.

$$\frac{\partial E[v|Z^{Cvs}]}{\partial \delta} > \frac{\partial E[v|Z^{Cvs}]}{\partial \delta} \Leftrightarrow$$

$$\{v'(U + s_G + \delta) - v'(U + s_G - \delta) - v'(L - 0.5\delta) + v'(L + 0.5\delta)\} \qquad \text{[Eq. A.4]}$$

$$> \{v'(L + s_B + \delta) - v'(L + s_B - \delta) - v'(U - 0.5\delta) + v'(U + 0.5\delta)\}$$

Next, consider the problem if we let $U = L$. In this case, the FOC of equations A.2.a and A.2.b imply $s_G = s_B$ holds. This means that, with $U = L$, A.4 will hold with equality. Our next step is to investigate A.4 as we increase U. Differentiating each side of A.4 with respect to U gives the following equations.

$$\frac{\partial^2 E[v|Z^{Cvs}]}{\partial \delta \partial U} = 0.25 \times \{v''(U + s_G + \delta) - v''(U + s_G - \delta)\} + \frac{d^2 E[v|z^{Csv}]}{d\delta ds_G} \frac{ds_G}{dU} \qquad \text{[Eq. A.5.a]}$$

$$\frac{\partial^2 E[v|Z^{Lib}]}{\partial \delta \partial U} = 0.25 \times \{-v''(U - 0.5\delta) + v'(U + 0.5\delta)\} + \frac{d^2 E[v|z^{Lib}]}{d\delta ds_B} \frac{ds_B}{dU} \qquad \text{[Eq. A.5.b]}$$

From the FOC that showed $\dfrac{dE[v|z^{Csv}]}{ds_G} = 0 = \dfrac{dE[v|z^{Lib}]}{ds_B}$, it further follows that we also

have $\dfrac{d^2 E[v|z^{Csv}]}{d\delta ds_G} = 0 = \dfrac{d^2 E[v|z^{Lib}]}{d\delta ds_B}$.

To show that A.1 holds, it suffices to show that A.5.a exceeds A.5.b, or, rearranging and simplifying, it suffices to show that A.6 holds, where A.6 is given as follows.

$$\frac{\partial^2 E[v|z^{Csv}]}{\partial \delta \partial U} > \frac{\partial^2 E[v|z^{Lib}]}{\partial \delta \partial U} \Leftrightarrow$$

$$\{v''(U + s_G + \delta) - v''(U + s_G - \delta)\} > \{-v''(U - 0.5\delta) + v''(U + 0.5\delta)\} \qquad \text{[Eq. A.6]}$$

$$\Leftrightarrow 0 > \{v''(U + s_G - \delta) - v''(U - 0.5\delta)\}$$

$$+ \{v''(U + 0.5\delta) - v''(U + s_G + \delta)\}$$

The final steps in the proof show that, for prudent DM's, equation A.6 holds by showing that the two difference on the right hand side of the final inequality are both negative.

First, we show that $0.5\delta > s_G > 0$. Since the DM is prudent, by assumption, we know that $s_G > 0$. To show $0.5\delta > s_G$ hold, suppose that $0.5\delta = s_G$. Substituting into the FOC equation A.2.a, we get the following.

$$-v'(U - 0.5\delta) + 0.5 \times \{v'(U + 1.5\delta) + v'(U - 0.5\delta)\} < 0 \qquad \text{[Eq. A.7]}$$

Hence, it must be that $0.5\delta > s_G$ to increase the left hand side of equation a.7. But since the DM is prudent, we know that is convex, so is increasing, and by risk aversion, is negative. Hence the following are true.

$$\{v''(U + s_G - \delta) - v''(U - 0.5\delta)\} < 0 \qquad \text{[Eq. A.8.a]}$$

$$\{v''(U + 0.5\delta) - v''(U + s_G + \delta)\} < 0 \qquad \text{[Eq. A.8.b]}$$

This shows that equation A.6 holds, completing the proof of Theorem 1.

Corollary 1.1: Under conditions of Theorem 1, if for all savings levels, $h(s) > 0$ and $h'(s) < 0$ both hold, then $Z_{Csv} \succ Z_{Lib}$ holds.

Proof of Corollary 1.1:

Corollary 1.1 follows immediately from Theorem 1 when we use the prior result R1. Prior results R1 says that $h'(s) < 0$ implies $\eta(s) > h(s)$. Since by assumption

$h(s) > 0$, it follows that $\eta(s) > h(s)$, which implies that $Z_{Csv} \succ Z_{Lib}$, completing the proof

of corollary 1.1.   Q.E.D. on Corollary 1.1.

Theorem 2: Let the conditions of Theorem 1 hold, suppose $\eta(s) > 0, \forall s$, and also

now assume each accounting system costs the DM a cost of $C > 0$ per signal generated.

Then there exists cut-off costs, $0 < C_1 < C_2$, such that the following are true.

 a. For $0 < C < C_1$, $Z^{Full} \succ Z^{Csv}$ holds.

 b. For $C = C_1$, $Z^{Full} \approx Z^{Csv}$ holds.

 c. For $C_1 < C < C_2$, $Z^{Csv} \succ Z^{Full}$ holds.

 d. For $C = C_2$, $Z^{Csv} \approx Z^{Part}$ holds.

 e. For $C_2 < C$, $Z^{Part} \succ Z^{Csv}$ holds.

Proof of theorem 2:

We begin the proof by first providing some additional equations for expected

utility with under different signals. Let $D(U)$ and $ND(U)$ be the expected utility when

earnings are good under full and partial disclosure, respectively, so that $D(U)$ and

$ND(U)$ are given as follows.

$$D(U) \equiv 0.5 \times E\left[v \big| z_1^{Full}\right] + 0.5 \times E\left[v \big| z_2^{Full}\right] = 0.5v(U - 0.5\delta) + 0.5v(U + 0.5\delta)$$
$$ND(U) \equiv \max_{s_G} E[v|G] = \max_{s_G}\left[v(U - s_G) + 0.5 \times \{v(U + s_G + \delta) + v(U + s_G - \delta)\}\right]$$

Let $D(L)$ and $ND(L)$ be the expected utility when earnings are good under full and

partial disclosure, respectively, so that $D(L)$ and $ND(L)$ are given as follows.

$$D(L) \equiv 0.5 \times E\left[v \big| z_3^{Full}\right] + 0.5 \times E\left[v \big| z_4^{Full}\right] = 0.5v(L - 0.5\delta) + 0.5v(L + 0.5\delta)$$
$$ND(L) \equiv \max_{s_B} E[v|B] = \max_{s_B}\left[v(L - s_B) + 0.5 \times \{v(L + s_B + \delta) + v(L + s_B - \delta)\}\right]$$

We see that $D'(U) > 0$, $ND'(U) > 0$, $D(U) - ND(U) > 0$, and $D'(U) - ND'(U) > 0$,

where the final inequality follows if $v'(\cdot)$ is convex, which holds if the DM is prudent,

while similar inequalities hold for bad news.

Without loss of generalization, we assume partial recognition has zero cost, liberal and conservative accounting have a cost of $C > 0$ and full recognition has a cost of $2C > 0$. Since $D(L) - ND(L) > 0$, the expected utility function are monotonic increasing and concave, there exist a cost, $C_2 > 0$, at which the investor is indifferent between conservative accounting and partial recognition. This means the following equation holds.

$$ND(L) + ND(U) = D(L - C_2) + ND(U - C_2) \qquad \text{[Eq. A.9]}$$

Concavity ensures that the expected utility under conservatism must cross the constant level of expected utility from the partial recognition system for some cost sufficient low. This shows that part d holds, while part e. follows immediately. Also, we know that for sufficiently low cost $C > 0$, we have $Z^{Full} \succ Z^{Csv}$, so part a hold. Hence, to complete the proof, we need only show that parts b. and c. hold, i.e., that there exists $C = C_1$ such that $Z^{Full} \approx Z^{Csv}$ holds and that for $C_1 < C < C_2$, that $Z^{Csv} \succ Z^{Full}$ holds.

We have shown that the expected utility under both conservatism and full recognition is decreasing concave functions of cost. Hence, since part a holds, we need only show that there exists a $C > 0$ where $Z^{Csv} \succ Z^{Full}$ holds. We do this by constructing the following argument. Suppose, at $C = C_2$, the DM is currently observing signals reported under conservatism, but is given the opportunity to change how the signals are reported for good news. He is given the opportunity to adopt the following uncertain system: receive a perfect signal with probability $\tau > 0$ and to continue to receive the imperfect signal with probability $1 - \tau$. He can make this change at a cost of $\tau C_2 > 0$. If we can show that the expected utility of this opportunity is negative for $\tau > 0$, this implies that $Z^{Csv} \succ Z^{Full}$ holds at $C = C_2$, completing the proof.

Consider the expected utility for the DM under the proposed new uncertain accounting system as a function of the probability $\tau > 0$. This expected utility, denoted as $W(\tau)$, is given as follows.

$$W(\tau) = D(L - C_2(1 + \tau)) + (1 + \tau)ND(U - C_2(1 + \tau)) + \tau D(L - C_2(1 + \tau)) \qquad \text{[Eq. A.10]}$$

We see that for $\tau = 0$, the expected utility equals the expected utility under conservatism, while for $\tau = 1$, the expected utility equals the expected utility under full recognition. Hence, as mentioned earlier, if we show $W'(\tau) < 0$, this suffices to show that

$Z^{Csv} \succ Z^{Full}$ holds at $C = C_2$.

Taking the derivative of equation A.10 with respect to τ, we have the following equation.

$$
\begin{aligned}
W'(\tau) &= \left[D(U - C_2(1+\tau)) - ND(U - C_2(1+\tau)) \right] \\
&\quad - C_2 \left[D'(L - C_2(1+\tau)) + (1+\tau)ND'(U - C_2(1+\tau)) + \tau D'(U - C_2(1+\tau)) \right] \\
&< \left[D(L - \tau C_2) - ND(L - \tau C_2) \right] \\
&\quad - C_2 \left[D'(L - C_2(1+\tau)) + (1+\tau)ND'(U - C_2(1+\tau)) + \tau D'(U - C_2(1+\tau)) \right] \\
&= \left[D(L - \tau C_2) + ND(L) + ND(U) - D(L - C_2) - ND(U - C_2) - ND(L - \tau C_2) \right] \\
&\quad - C_2 \left[D'(L - C_2(1+\tau)) + (1+\tau)ND'(U - C_2(1+\tau)) + \tau D'(U - C_2(1+\tau)) \right]
\end{aligned}
$$

[Eq. A.11]

The inequality followed from the observations that $D'(U) - ND'(U) > 0$ and $L < U - C_2$. We used equation A.9 for the final equality.

Next, we use the concavity of the expected utility functions to produce two useful inequalities. In general, for any increasing, concave function $g(x)$, for $y \leq x$ we have the following inequality.

$$-kg(y) + \left[g(x) - g(x - k) \right] < 0$$

Hence, the concavity of $D(U)$ and $ND(U)$ ensure that the following inequalities hold.

$$-C_2(1-\tau)\left[D'(L - C_2(1+\tau)) \right] + \left[D(L - \tau C_2) - D(L - C_2) \right] < 0 \qquad \text{[Eq. A.12.a]}$$

$$-C_2(1-\tau)\left[ND'(U - C_2(1+\tau)) \right] + \left[ND(U) - ND(U - C_2) \right] < 0 \qquad \text{[Eq. A.12.b]}$$

We make the following substitutions in the equation to simplify subsequent derivations.

$$A_1 = \left[D'(L - C_2(1+\tau)) + ND'(U - C_2(1+\tau)) \right]$$
$$A_2 = \left[D'(U - C_2(1+\tau)) + ND'(U - C_2(1+\tau)) \right]$$

Using these variables equation A.11 can be rewritten as follows.

$$
\begin{aligned}
W'(\tau) &< -C_2 \left[A_1 + \tau A_2 \right] \\
&\quad + \left[D(L - \tau C_2) + ND(L) + ND(U) - D(L - C_2) - ND(U - C_2) - ND(L - \tau C_2) \right] \\
&= -\tau C_2 \left[A_1 + A_2 \right] - (1-\tau)C_2 A_1 \\
&\quad + \left[D(L - \tau C_2) + ND(L) + ND(U) - D(L - C_2) - ND(U - C_2) - ND(L - \tau C_2) \right]
\end{aligned}
$$

[Eq. A.13]

Using equations A.12.a and A.12.b, we have the following inequality.

$$-(1-\tau)C_2A_1 + [D(L-\tau C_2) + ND(U) - D(L-C_2) - ND(U-C_2)] < 0 \qquad \text{[Eq. A.14]}$$

Substituting into A.13 using equation A.14, we get the following inequality.

$$\begin{aligned} W'(\tau) &< -\tau C_2[A_1 + A_2] + [ND(L) - ND(L-\tau C_2)] \\ &= -\tau C_2[D'(L-C_2(1+\tau)) + D'(U-C_2(1+\tau))] \qquad \text{[Eq. A.15]} \\ &+ [ND(L) - ND(L-\tau C_2)] < 0 \end{aligned}$$

The final inequality holds at $\tau = 0$, completing the proofs of parts b and c and

completing the proof of Theorem 2.

Corollary 2.1: Under conditions of Theorem 1, the DM is offered an opportunity

to adopt a partial recognition system with costly auditing, with perfect information

revealed under good and bad news with probability and , respectively. Then DM's having

a non-zero prudence measure would never prefer a neutral system. Instead, for

intermediate cost levels, DM's prefer systems with higher probability on recognizing bad

news or good news, as they are prudent or imprudent, respectively. I.e., $\eta(s) \neq 0$ implies

$0 < \gamma_B = \gamma_G < 1$ is never optimal, while $\eta(s) > 0$ and $\eta(s) < 0$ implies $\gamma_B > \gamma_G$ and

$\gamma_B > \gamma_G$ is preferred, respectively.

Proof of Corollary 2.1:

First, let $Z^{Aud}(\gamma_B, \gamma_G)$ denote a partial recognition system with costly auditing

under probabilities $0 < \gamma_B = \gamma_G < 1$. Using the notation introduced in the proof of

Theorem 2, the expected utility at date 0 under system $Z^{Aud}(\gamma_B, \gamma_G)$ is given as follows.

$$\begin{aligned} E[v|Z^{Aud}(\gamma_B, \gamma_G)] &= .05[\gamma_B D(L-C) + (1-\gamma_B)ND(L-C)] \\ &+ .05[\gamma_G D(U-C) + (1-\gamma_G)ND(U-C)] \end{aligned} \qquad \text{[Eq. A.16]}$$

Next, let $0 < \gamma_B = \gamma + \tau < 1$ and $0 < \gamma_G = \gamma - \tau < 1$, so that we start with a neutral system

(with $0 = \tau$) and consider the impact of shifting probability on recognition from good

news to bad news or vice versa. Taking the derivative of the expected utility with respect

to $0 = \tau$, we get the following equation.

$$.05[D(L-C) - ND(L-C)] - 0.5[D(U-C) - ND(U-C)] \qquad \text{[Eq. A.17]}$$

Equation A. 16 will be positive as $D(U) - ND(U)$ is decreasing (or increasing), indicating that prudent (or imprudent) DM's will prefer to increase (or decrease) $\gamma_B = \gamma + \tau$ relative to $\gamma_G = \gamma - \tau$, while neither will wish to keep $0 = \tau$, completing the proof of corollary 2.1. Q.E.D. on Corollary 2.1.

Theorem 3: For all expected utility maximizing DM's facing the precautionary savings problem described in P1 above, the follow are true.

a. Decreasing risk aversion is not necessary to ensure that the DM prefers the conservative system, i.e., there exists a utility function with where $Z^{Csv} \succ Z^{Lib}$ and $h'(s) > 0, \forall s$ both hold.

b. Risk aversion is not sufficient to insure that the DM prefers a conservative accounting system, i.e., there exists a utility function where $Z^{Csv} \prec Z^{Lib}$ and $h(s) > 0, \forall s$ both hold.

Proof of Theorem 3:

We use examples based on the utility function, $v(x) = a - b(k - x)^{\gamma}$ to prove both parts. For part a. we use $\gamma = 3$ while for part b. we use $\gamma = 1.5$. See appendix B below for the detail for these cases.

8. APPENDIX B Presentation of examples

In this appendix we present some examples of common utility functions and the relative preferences for conservative versus liberal accounting information systems.

Case 1: Natural logarithm utility function.

We start with the natural logarithm utility function, which is a function that exhibits both absolute risk aversion everywhere as well as decreasing absolute risk aversion (DARA).

$v(x) = \ln x$; $v'(x) = x^{-1}$; $v''(x) = -x^{-2}$; $v'''(x) = 2x^{-3} > 0$ always, so $v'(x)$ is convex

$h(x) = $ Risk Aversion $= x^{-1;}$ Risk aversion is always decreasing; DARA

$\eta(x) = $ Prudence $= 2x^{-1} > h(x) = $ Risk Aversion $x^{-1} > 0$. Conservative accounting
is always preferred.

Case 2: Negative exponential utility function.

We next turn to the negative exponential utility function, one of the most common utility functions used in economic theory. This has the useful characteristic that it exhibits constant absolute risk aversion (CARA), so $h(x)$ is constant, and also exhibits constant prudence, where the prudence and absolute risk aversion measures are equal.

$v(x) = -e^{-\gamma x}$; $v'(x) = \gamma e^{-\gamma x}$; $v''(x) = -\gamma^2 e^{-\gamma x}$; $v'''(x) = \gamma^3 e^{-\gamma x} > 0$ always; So $v'(x)$ is
convex.

$\alpha(x) = $ Risk Aversion $= \gamma$ Risk aversion is constant; CARA

$\eta(x) = $ Prudence $= \gamma = \alpha(x) = $ Risk Aversion > 0. Conservative accounting is
always preferred.

Cases 3a and b: Positive exponent utility functions.

We next turn to a more complicated sequence of utility functions based on taking a positive exponent on the consumption variable. We start with a simple case, where the

utility function composed solely of the consumption variable raised to a positive exponent less than 1. We follow with more general functions.

Case 3.a: Simple positive exponent utility function.

$v(x) = x^\gamma.$ $0 < \gamma < 1$; $v'(x) = \gamma x^{\gamma-1}$; $v''(x) = \gamma(\gamma-1)x^{\gamma-2}$; $v'''(x) = \gamma(\gamma-1)(\gamma-2)x^{\gamma-3.} > 0,$

always; So $v'(x)$ is convex.

$\alpha(x)$ = Risk Aversion = $-(\gamma-1)/x > 0$; Risk aversion is decreasing; DARA

$\eta(x)$ = Prudence = $-(\gamma-2)/x > \alpha(x)$ = Risk Aversion = $-(\gamma-1)/x > 0$

Conservative accounting is always preferred.

Case 3.b: General form of positive exponent utility function.

$v(x) = K^\gamma - (K-x)^\gamma.$ $1 < \gamma$; *defined only for* $x < K$

$v'(x) = \gamma(K-x)^{\gamma-1}$; $v''(x) = -\gamma(\gamma-1)(K-x)^{\gamma-2}$; $v'''(x) = \gamma(\gamma-1(\gamma-2))(K-x)^{\gamma-3.} > 0$ *only*

if $\gamma > 2$; So $v'(x)$ is convex only if $\gamma > 2$ (i.e., prudent), otherwise concave (i.e., imprudent) and linear if $\gamma = 2$ (i.e., aprudent).

$\alpha(x)$ = Risk Aversion = $(\gamma-1)/(K-x) > 0$; Risk aversion is always increasing;

IARA

$\eta(x)$ = Prudence = $(\gamma-2)/(K-x) < \alpha(x)$ = Risk Aversion = $-(\gamma-1)/x > 0.$

Again, as noted above, prudence > 0 if $\gamma > 2$, then Conservative accounting is preferred; prudence < 0 if $\gamma < 2$, then Conservative accounting is not preferred; and prudence $= 0$ if $\gamma = 2$, and then investor is prudent neutral or is indifferent between Conservative and Liberal accounting.

Even if we have IARA, Conservative accounting is preferred as long as $\gamma > 2$, so that prudence > 0.